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The investigators developed new mathematical control methods and theory for nonlinear systems with time delays and uncertainty that included new controller designs and robustness analysis, adaptive control with parameter identification, and robust forward invariance that maintained state constraints under controller uncertainty and time delays. They also developed new optimal control and optimization theory for differential inclusions based on semiconcavity, state constraints, and systems on stratified domains. The investigators applied their theory to UAVs, planar vertical takeoff and landing aircraft, curve tracking using gyroscopic controls, and other important engineering models. The investigators authored 28 project-related archival publications that appeared or were accepted for publication or submitted during the project period. They supervised two project-related dissertations, one on Hamilton-Jacobi theory for stratified systems and a second on aerospace models with input constraints. The investigators also presented 53 conference or seminar talks. In recognition of their research, PI Malisoff was appointed as the Roy Paul Daniels Profes

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Final Report for AFOSR Grant FA9550-09-1-0400

The investigators developed new mathematical control methods and theory for nonlinear systems with time delays and uncertainty that included new controller designs and robustness analysis, adaptive control with parameter identification, and robust forward invariance that maintained state constraints under controller uncertainty and time delays. They also developed new optimal control and optimization theory for differential inclusions based on semiconcavity, state constraints, and systems on stratified domains. Semiconcavity helps form the basis for efficient numerical schemes, such as those by McEneaney that do not suffer from the curse of dimensionality. Stratified domains help capture hybrid settings where the system switches between different modes of operation by moving between different regions of the state space.

The part of the project dealing with stratified systems gave invariance conditions for keeping the dynamics in certain desired sets, and characterizations of the minimal cost function as the solution of a Hamilton-Jacobi equation that satisfies certain side conditions [B10], which have the potential to help develop computational methods involving the minimal cost function. The work [CMW12b] found new relationships involving the Pontryagin maximum principle and the value function from dynamic programming which introduce duality. The PIs anticipate that this work will be useful for numerical calculations involving the value function and Hamilton-Jacobi equations, such as those that were done by Falcone, Krener, McEneaney, and other AFOSR sponsored researchers. The work [WZ12] replaces Soner's pointing condition by a new set of tangential conditions where the dimension of the state constraint can change, which can potentially include network flow problems. The reflected control problems in [SWZ12] are similar to state constrained problems except the dynamics are modified to stay in the state constraint set, and instead of the classical pointing condition, the dynamics can run along the boundary on a smaller dimensional manifold.

The investigators applied their theory to UAVs, planar vertical takeoff and landing (PVTOL) aircraft, curve tracking using gyroscopic controls, and other important engineering models. The significance of the PVTOL results [GMM11,GMM12b] is in (a) the global boundedness of the controllers in the decoupled coordinates, (b) their applicability to cases where the velocity measurements may be unavailable, when the original system is augmented by an observer for the unknown velocities, (c) the uniform global asymptotic stability of the tracking dynamics, (d) the applicability of the theory to a very general class of reference trajectories, and (e) the use of input-to-state stability to quantify performance under actuator errors of large amplitude. The work for UAVs in [GMM12a,GMM13,MM12a,MM12b] gave globally bounded tracking controllers under mild nondegeneracy conditions on the reference trajectory and the reference control, including (i) input-to-state stability with respect to small additive uncertainty on the controllers and (ii) compensation of arbitrarily long time delays in the state observations using Lyapunov-Krasovskii functionals. The UAV work can be applied when there are rate constraints on the controls (which are constant bounds on the rate of change of the controller values along all of the system trajectories) and can ensure that the UAV velocity remains above a desired minimal value. The primary delay compensation strategy in the project consisted of building controllers that are the sum of two terms. The first term has no delay. The second term has the delay and can be made arbitrarily small in amplitude without sacrificing the stability of the closed loop system. By tuning the parameters in the second term, one can ensure tracking

under any positive constant delay. The PIs also applied their delay systems methods to biroeactors in [DDWW12,MM10b,MM10c], and they anticipate that many other important applications of their work may be possible.

The investigators authored 28 project-related archival publications that appeared or were accepted for publication or submitted during the project period. They also supervised two project-related dissertations in the LSU mathematics department, one on Hamilton-Jacobi theory for stratified systems [B10], and the other on bounded backstepping and tracking for aerospace models under input constraints [G12]. The investigators also presented 53 conference or seminar talks and won two Best Presentation Awards in American Control Conference sessions during the project period. Graduate Assistant Aleksandra Gruszka was one of the five finalists for the Student Best Paper Award at the 2011 American Control Conference where she also won a best presentation award in her session, and she was one of the 12 US graduate students selected to present their work at the Association for Women in Mathematics sessions at the 2012 Joint Mathematics Meetings. In recognition of their research, PI Malisoff was appointed as the Roy Paul Daniels Professor #3 in the LSU College of Science, and PI Wolenski received a lifetime appointment as the Russell B. Long Professor of Mathematics.

Project-Related Publications

See https://www.math.lsu.edu/research/controltheory for links to articles and presentations.

[B10] Barnard, R., Hamilton-Jacobi Theory for Optimal Control Problems on Stratified Domains, *Ph.D. Dissertation*, Department of Mathematics, Louisiana State University, Baton Rouge, LA, 2010. etd.lsu.edu/

[BW12] Barnard, R., and P. Wolenski, "Flow invariant properties for stratified systems," submitted.

[CMW12a] Cannarsa, P., F. Marino, and P. Wolenski, "Semiconcavity of the minimum time function for differential inclusions," *Dynamics of Continuous, Discrete and Impulsive Systems. Series B. Applications and Algorithms*, Volume 19, Numbers 1-2, 2012, pp. 187-206.

[CMW12b] Cannarsa, P., F. Marino, and P. Wolenski, "Adjoint inclusions for differential inclusions," submitted.

[CW11] Cannarsa, P., and P. Wolenski, "Semiconcavity of the value function for a class of differential inclusions," *Discrete and Continuous Dynamical Systems-Series A*, Volume 29, Number 2, 2011, pp. 453-466.

[DDWW12] Drame, A., D. Dochain, J. Winkin, and P. Wolenski, "Periodic trajectories of distributed parameter biochemical systems with time delay," *Applied Mathematics and Computation*, Volume 218, Issue 14, 2012, pp. 7395-7405.

[G12] Gruszka, A., Some Tracking Problems for Aerospace Models with Input Constraints, *Ph.D. Dissertation*, Department of Mathematics, Louisiana State University, Baton Rouge, LA, 2012. etd.lsu.edu/

[GMM11] Gruszka, A., M. Malisoff, and F. Mazenc, “On tracking for the PVTOL model with bounded feedbacks,” in *Proceedings of the 2011 American Control Conference (San Francisco, CA, 29 June-1 July, 2011),* pp. 1428-1433.

[GMM12a] Gruszka, A., M. Malisoff, and F. Mazenc, “Tracking and robustness analysis for UAVs with bounded feedbacks,” in *Proceedings of the 2012 American Control Conference (Montreal, Canada, 27-29 June 2012)*, pp. 932-937.

[GMM12b] Gruszka, A., M. Malisoff, and F. Mazenc, “Tracking control and robustness analysis for PVTOL aircraft under bounded feedbacks,” *International Journal of Robust and Nonlinear Control*, to appear.

[GMM13] Gruszka, A., M. Malisoff, and F. Mazenc, “Bounded tracking controllers and robustness analysis for UAVs,” *IEEE Transactions on Automatic Control*, Volume 58, Number 1, January 2013, to appear.

[GW12] Guevara, A., and P. Wolenski, “Convergence results for a self-dual regularization of convex problems,” *Optimization*, Volume 61, Number 6, 2012, pp. 699-716.

[MalMaz09] Malisoff, M., and F. Mazenc, <u>Constructions of Strict Lyapunov Functions</u>, Communications and Control Engineering Series, Springer-Verlag London Ltd., London, UK, 2009.

[MMZ11] Malisoff, M., F. Mazenc, and F. Zhang, “Input-to-state stability for curve tracking control: A constructive approach,” in *Proceedings of the 2011 American Control Conference (San Francisco, CA, 29 June-1 July, 2011)*, pp. 1984-1989.

[MMZ12] Malisoff, M., F. Mazenc, and F. Zhang, “Stability and robustness analysis for curve tracking control using input-to-state stability,” *IEEE Transactions on Automatic Control*, Volume 57, Number 5, May 2012, pp. 1320-1326.

[MZ12a] Malisoff, M., and F. Zhang, “Adaptive controllers and robustness analysis for curve tracking with unknown control gains,” in *Proceedings of the 2012 American Control Conference (Montreal, Canada, 27-29 June 2012)*, pp. 344-349.

[MZ12b] Malisoff, M., and F. Zhang, “Adaptive control for planar curve tracking under controller uncertainty,” submitted.

[MazMal09] Mazenc, F., and M. Malisoff, “Lyapunov functions under LaSalle conditions with an application to Lotka-Volterra systems,” in *Proceedings of the American Control Conference (St. Louis, MO, 10-12 June 2009)*, pp. 96-101.

[MM10a] Mazenc, F., and M. Malisoff, “Strict Lyapunov function constructions under LaSalle conditions with an application to Lotka-Volterra systems,” *IEEE Transactions on Automatic Control*, Volume 55, Number 4, April 2010, pp. 841-854.

[MM10b] Mazenc, F., and M. Malisoff, “Stabilization of two-species chemostats with delayed measurements and Haldane growth functions,” in *Proceedings of the American Control Conference (Baltimore, MD, 30 June-2 July 2010)*, pp. 6740-6744.

[MM10c] Mazenc, F., and M. Malisoff, “Stabilization of a chemostat model with Haldane growth functions and a delay in the measurements,” *Automatica*, Volume 46, Issue 9, September 2010, pp. 1428-1436.

[MM12a] Mazenc, F., and M. Malisoff, “Stabilization for feedforward systems with delay in the input,” in *Proceedings of the 51st IEEE Conference on Decision and Control (Maui, HI, 10-13 December 2012)*, to appear.

[MM12b] Mazenc, F., and M. Malisoff, “Asymptotic stabilization for feedforward systems with delayed feedbacks,” submitted.

[MMdQ10] Mazenc, F., M. Malisoff, and M. de Queiroz, “On uniform global asymptotic stability of adaptive systems with unknown control gains,” in *Proceedings of the American Control Conference (Baltimore, MD, 30 June-2 July 2010)*, pp. 166-171.

[MMdQ11] Mazenc, F., M. Malisoff, and M. de Queiroz, “Uniform global asymptotic stability of adaptive cascaded nonlinear systems with unknown high-frequency gains,” *Nonlinear Analysis Theory Methods and Applications*, Volume 74, Issue 4, February 2011, pp. 1132-1145.

[MMD12] Mazenc, F., M. Malisoff, and T. Dinh, “Robustness of nonlinear systems with respect to delay and sampling of the controls,” submitted.

[SWZ12] Serea, O., P. Wolenski, and H. Zidani, “Lower semicontinuous solutions to reflected control problems,” submitted.

[WZ12] Wolenski, P., and H. Zidani, “A new approach to state constrained optimal control problems,” submitted.